

Multiphase Flow and Heat Transfer

ME546

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Condensation

- Condensation is a process in which the removal of heat from a system causes a vapor to convert into liquid.
- The spectrum of flow processes associated with condensation on a solid surface is almost a mirror image of those involved in boiling.
- Can also occur on a free surface of a liquid or even in a gas
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- Condensation processes are numerous, taking place in a multitude of situations.
- Important role in nature:
 - Crucial component of the water cycle
 - Industry

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1. Mode of condensation:
 1. Homogeneous
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3. System geometry: plane surface, external, internal, etc.

There are overlaps among different classification methods.

Classification based on mode of condensation is the most useful.

Homogeneous Condensation

- Can happen when vapor is sufficiently cooled $< T_{sat}$ to induce droplet nucleation.
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 - Cloud - water or ice? -30°C
- Although homogeneous nucleation in pure vapors is possible, in practice dust, other particles act as droplet nucleation embryos

Heterogeneous Condensation

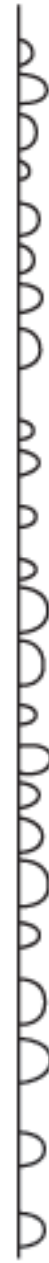
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Heterogeneous Condensation

- Droplets form and grow on solid surfaces
- Significant sub-cooling of vapor is required for condensation to start when the surface is smooth and dry.
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- Heterogeneous condensation leads to:
 1. Dropwise condensation
 2. Film condensation



Film Condensation



Dropwise Condensation

Dropwise vs Film

- **Film:** The surface is blanketed by a liquid film of increasing thickness. “Liquid wall” offers resistance.
- **Dropwise:** The droplets slide down when they reach a certain size, clearing the surface and exposing it to vapor.
- No resistance to heat transfer in dropwise. Hence, h is 10 times higher than in film.

Dropwise Condensation

- Drop condensation on the underside of a cooled horizontal plate or on a vertical surface is very analogous to nucleate boiling. Ex: misting up of windows or mirrors.
- The condensate forms droplets that stick to the surface.
- The population of droplets becomes large, run together to form films – transition to film condensation.



Dropwise Condensation

- Poorly wetted surface: on a solid surface cooled below T_{sat}
- At locations of well-wetted: contaminant nuclei exist.
- Poorly-wetted surface condition: contamination or coating with a hydrophobic substance.
- Droplets grow, fall off, new droplets on freshly exposed surface.
- Sweeping and renewal of the droplet growth process is responsible for the high h .



Dropwise Condensation

- In practice, this can be achieved from steam condensation by
 1. Injecting a non-wetting chemical into the vapor which subsequently deposits on the surface
 2. Introducing a non-wetting ($\theta > 90^\circ$) substance such as a fatty acid or wax onto the solid surface
 3. By permanently coating the surface with a low-surface-energy polymer or a noble metal.
- The effects of 1 & 2 are temporary.

Dropwise Condensation

- Providing and maintaining the non-wetting surface characteristics can be difficult.
- The condensate liquid often gradually removes the promoters.
- Furthermore, the accumulation of droplets on a surface can eventually lead to the formation of a liquid film.

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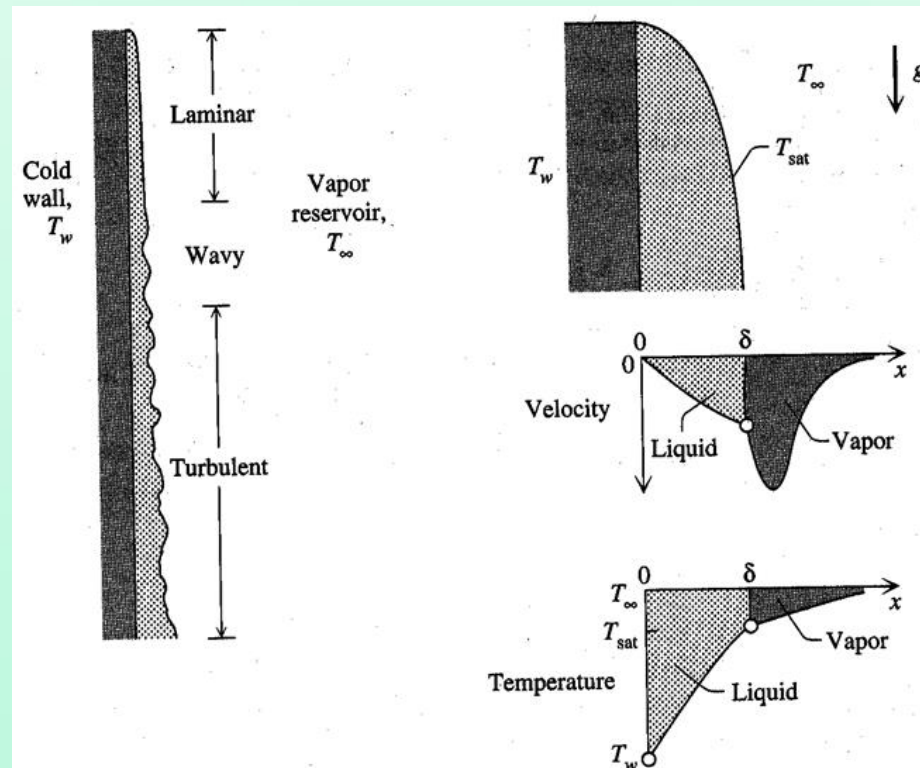
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For a surface subcooling of 8 K, determine the dropwise heat transfer coefficient of saturated steam at atmospheric pressure.

$$h_{dc} = 295 \text{ kW/m}^2\text{K}$$

Film Condensation on a Flat Vertical Surface

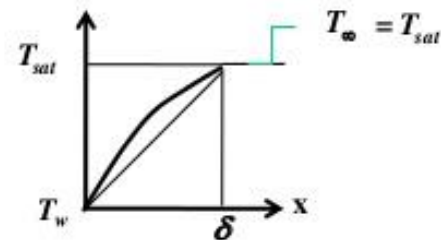
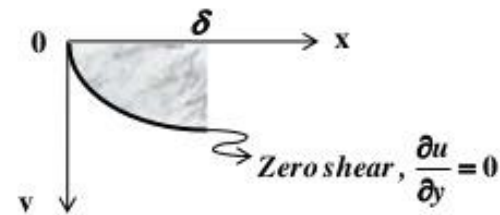
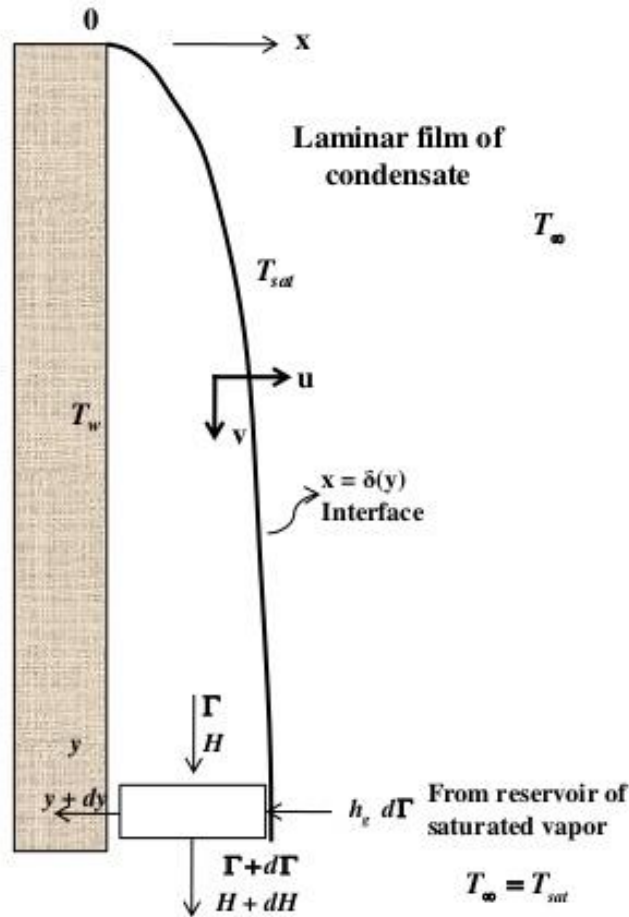


- Temperature of the liquid-vapor interface is the saturation temperature that corresponds to T_{sat}
- Vapor in the descending jet is colder than the vapor reservoir and warmer than the liquid in the film attached to the wall

Film Condensation on a Flat Vertical Surface

- The wall could be flat or outside surface of a vertical tube
- Consider a vertical wall exposed to a saturated vapor at pressure p and $T_{sat} = T_{sat}(p)$
- $T_w < T_{sat}$ vapor will continuously condense on the wall
- If the liquid wets the surface, liquid flows down the wall in a thin film
- Provided the condensation rate is not too large, there will be no discernable waves on the film surface, and the flow in the film will be laminar
 - Fluid dynamics of the flow of a thin liquid film
 - Heat transfer during the flow of a thin liquid film

Film Condensation on a Flat Vertical Surface



Nusselt Integral Analysis: Assumptions

- Laminar flow and constant properties
- Inertia effects are negligible in the momentum balance
- Gas is assumed to be pure vapor and at a uniform T_{sat}
 - This assumption allows us to focus exclusively on the flow of the liquid film and neglect the movement of the nearest layers of vapor
- Shear stress at the liquid-vapor interface is negligible
- With no temperature gradient in the vapor,
- Heat transfer to the liquid-vapor interface can occur only by condensation at the interface and not by conduction from the vapor

Steady State 2D Incompressible Flow

Continuity

$$\cancel{\frac{\partial u}{\partial x}} + \frac{\partial v}{\partial y} = 0 \quad \boxed{\frac{\partial v}{\partial y} = 0}$$

Momentum in x

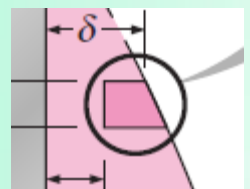
$$\rho_l \left(u \cancel{\frac{\partial u}{\partial x}} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial P}{\partial x} + \mu_l \left[\frac{\partial^2 u}{\partial x^2} + \cancel{\frac{\partial^2 u}{\partial y^2}} \right]$$

$$\boxed{\frac{\partial P}{\partial x} = 0 \Rightarrow P = P(y)}$$

Momentum in y

$$\rho_l \left(u \cancel{\frac{\partial v}{\partial x}} + v \cancel{\frac{\partial v}{\partial y}} \right) = - \frac{dP}{dy} + \mu_l \left[\frac{\partial^2 v}{\partial x^2} + \cancel{\frac{\partial^2 v}{\partial y^2}} \right] + \rho_l g$$

dP/dy = Pressure imposed from the inviscid portion
 = $\rho_v g$ = Hydrostatic pressure in vapor



$$\mu_l \frac{\partial^2 v}{\partial x^2} + (\rho_l - \rho_v)g = 0$$

Integrate for v

$$v = \frac{(\rho_l - \rho_v)g}{\mu_l} \delta^2 \left[\frac{x}{\delta} - \frac{1}{2} \left(\frac{x}{\delta} \right)^2 \right]$$

Boundary Conditions

$$x = 0, v = 0$$

$$x = \delta, \frac{\partial v}{\partial x} = 0$$

However, film thickness is an unknown function $\delta(y)$

The local mass flow rate of the condensate at a location y , where the boundary layer thickness is δ , is determined from:

$$\dot{m}(y) = \int_{x=0}^{\delta} \rho_l v b dx = \frac{\rho_l (\rho_l - \rho_v) g b \delta^3}{3 \mu_l}$$

The rate of condensation of vapor over a vertical distance dy

$$\frac{d\dot{m}}{dy} = \frac{\rho_l (\rho_l - \rho_v) g b \delta^2}{\mu_l} \frac{d\delta}{dy}$$

Flow rate is proportional to sinking effect, inversely proportional to μ

Steady State 2D Heat Transfer

Film velocity is low.

dT in y is negligible since both wall and film surface are isothermal

$$\frac{\partial^2 T}{\partial x^2} = 0$$

$$T = (T_{sat} - T_w) \frac{x}{\delta} + T_w$$

Boundary Conditions

$$x = 0, T = T_w$$

$$x = \delta, T = T_{sat}$$

This is a linear temperature profile similar to the conduction in a plane wall

Heat flux into the wall (k_l) = Heat flux across the film (h)

$$k_l \left. \frac{dT}{dx} \right|_w = h(T_{sat} - T_w)$$

$$h = \frac{k_l}{\delta}$$

Heat transfer across the film is by conduction alone

Film Thickness, δ

Rate of heat transfer from vapor to the plate through the liquid film dy
= Heat require to condense the vapor

$$k_l (b dy) \frac{T_{sat} - T_w}{\delta} = d\dot{m} h_{lv}$$

$$\therefore \frac{d\dot{m}}{dy} = \frac{\rho_l (\rho_l - \rho_v) g b \delta^2}{\mu_l} \frac{d\delta}{dy}$$

Solving for δ and integrating $\delta = (0, \delta)$ with $\delta = 0$ at $y = 0$

$$\delta(y) = \left[\frac{4\mu_L k_l (T_{sat} - T_w)}{\rho_L g (\rho_L - \rho_v) h_{lv}} y \right]^{\frac{1}{4}}$$

$$h = \frac{k_l}{\delta} = \left(\frac{\rho_l g (\rho_l - \rho_v) h_{lv} k_l^4}{4 \mu_l k_l (T_{sat} - T_w) y} \right)^{\frac{1}{4}}$$

$$\bar{h}_L = \frac{1}{L} \int_0^L h dy = \left(\frac{g \rho_l (\rho_L - \rho_v) h_{lv} k_l^3}{4 \mu_l (T_{sat} - T_w)} \right)^{\frac{1}{4}} \frac{1}{L} \int_0^L y^{-\frac{1}{4}} dy$$

$$\bar{h}_L = 0.943 \left(\frac{g \rho_l (\rho_L - \rho_v) h_{lv} k_l^3}{4 \mu_l (T_{sat} - T_w) L} \right)^{\frac{1}{4}}$$

$$\dot{m} = \frac{b \rho_l g (\rho_l - \rho_v)}{3 \mu_l} \left[\frac{4 \mu_l k_l^4 (T_{sat} - T_w)}{\rho_l g (\rho_l - \rho_v) h_{lv}} y \right]^{\frac{3}{4}}$$

All the properties are evaluated at: $T_f = \frac{T_{sat} + T_w}{2}$

Effect of subcooling

Rohsenow refined

- Avoided linear temperature profile
- Integral analysis of temperature distribution across the film
- Temperature profile whose curvature increases with the degree of subcooling $C_{p,l} (T_{sat} - T_w)$

$$h'_{lv} = h_{lv} + 0.68 C_{p,l} (T_{sat} - T_w)$$

All liquid properties evaluated at T_f

h_{lv} and ρ_v are evaluated at T_{sat}

$$h'_{lv} = h_{lv} (1 + 0.68 \text{Ja})$$

$$\text{Ja} = \frac{C_{p,l} (T_{sat} - T_w)}{h_{lv}}$$

Reynolds Number

$$\text{Re} = \frac{\rho_l u_m D_h}{\mu_l}; \quad u_m = \frac{\dot{m}}{\rho_L \delta}; \quad D_h = \frac{4A_c}{P} = \frac{4\delta b}{b} = 4\delta$$

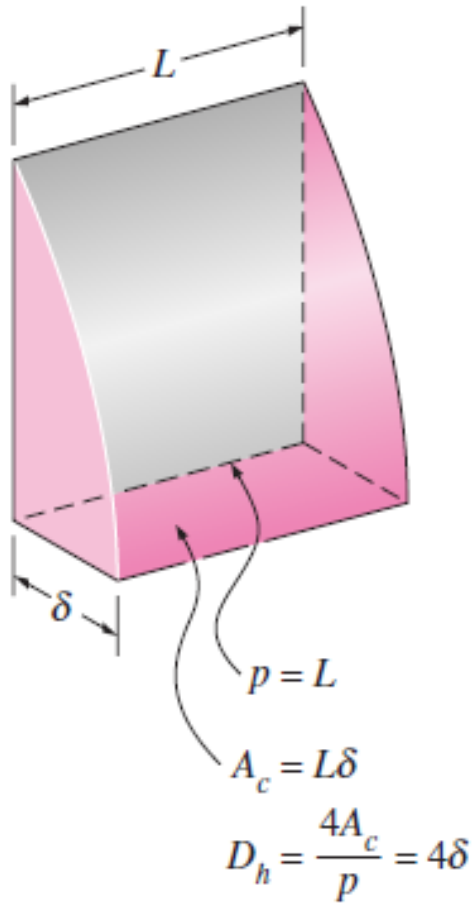
$$\text{Re} = \frac{4\dot{m}}{\mu_l}$$

$$\because \rho_L \gg \rho_v$$

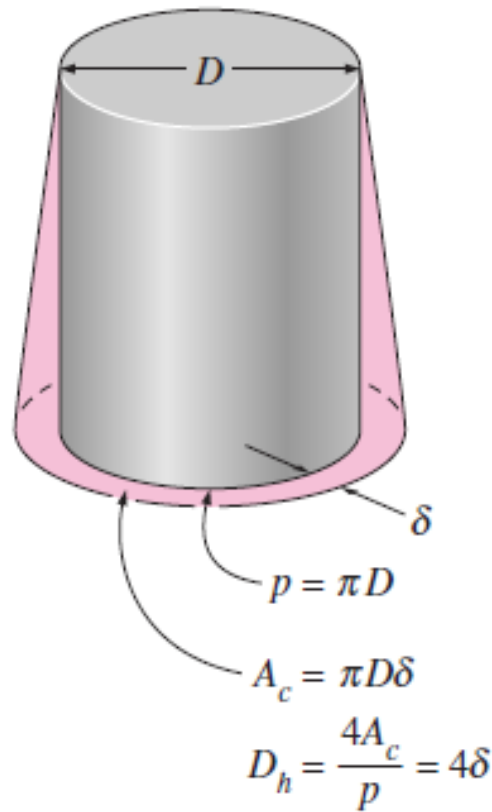
$$\text{Re} = \frac{4\rho_l^2 g \delta^3}{3\mu_l^2} = \frac{4g\delta^3}{3\nu_l^2}$$

$$h_{\text{avg}} = 1.47k_l \text{Re}^{-\frac{1}{3}} \left(\frac{g}{\nu_l^2} \right)^{\frac{1}{3}}$$

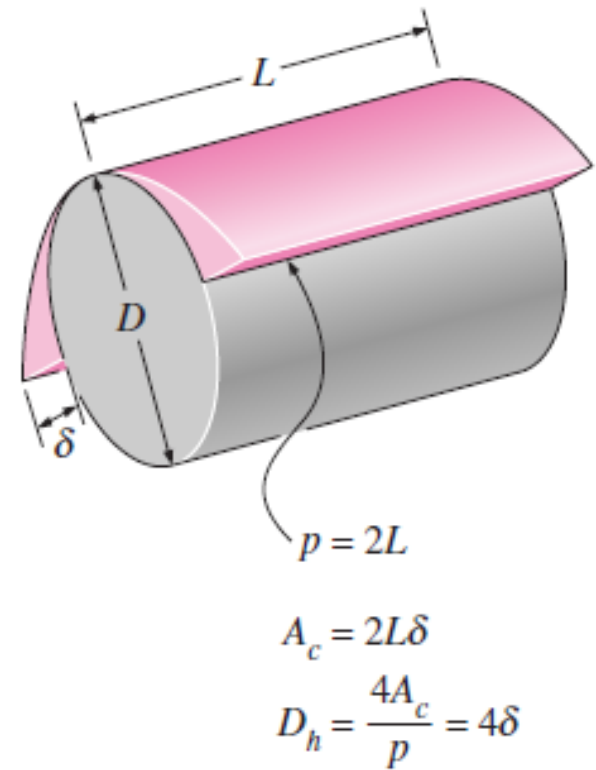
Hydraulic Diameter



(a) Vertical plate



(b) Vertical cylinder



(c) Horizontal cylinder

Wavy Laminar Flow over Vertical Plates

At $Re > 30$, waves form at the liquid vapor interface although the flow in liquid film remains laminar. The flow in this case is **Wavy Laminar**.

Kutateladze (1963) recommended the following relation for wavy laminar condensation over vertical plates:

$$h_{\text{vert,wavy}} = \frac{Re k_l}{1.08 Re^{1.22} - 5.2} \left(\frac{g}{v_l^2} \right)^{\frac{1}{3}}$$

$$30 < Re < 1800, \quad \rho_v \ll \rho_l$$

$$Re_{\text{vert,wavy}} = \left[4.81 + \frac{3.70 L k_l (T_{\text{sat}} - T_w)}{\mu_l h'_{fg}} \left(\frac{g}{v_l^2} \right)^{\frac{1}{3}} \right]^{0.82}$$

L is the height of the vertical plate

Turbulent Flow

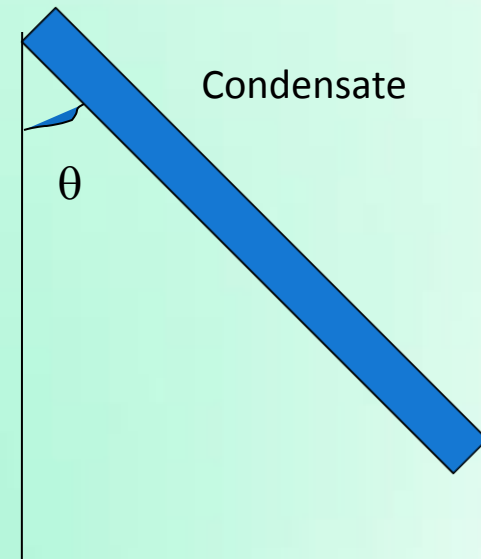
Vertical plates ($Re > 1800$)

$$h_{\text{vert,turbulent}} = \frac{Re k_l}{8750 + 58 Pr^{-0.5} (Re^{0.75} - 253)} \left(\frac{g}{\nu_l^2} \right)^{\frac{1}{3}}$$

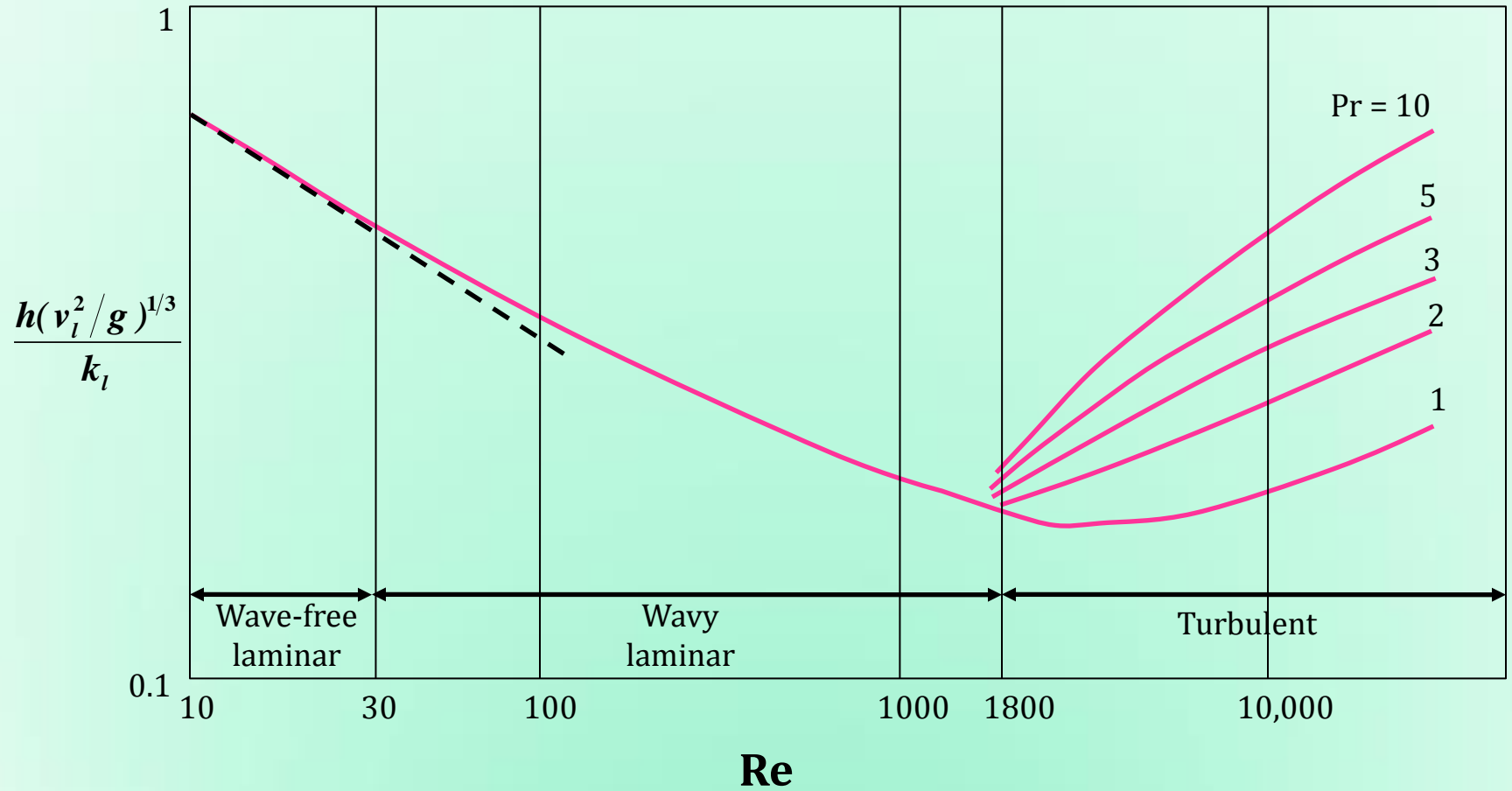
Film condensation on an inclined plate

$$h_{\text{inclined}} = h_{\text{vert}} \cos \theta$$

$$\frac{\bar{h}_L}{k_l} \left(\frac{\nu_l^2}{g} \right)^{\frac{1}{3}} = \left(Re_L^{-0.44} + (5.82 \times 10^{-6}) Re_L^{0.88} Pr_L^{\frac{1}{3}} \right)^{\frac{1}{2}}$$



Non-dimensional h for Condensation on Vertical Plates



Saturated steam at atmospheric pressure condenses on a 2 m high and 3 m wide vertical plate that is maintained at 80°C by circulating cooling water through the other side. Determine:

- (a) the rate of heat transfer by condensation to the plate
- (b) the rate at which the condensate drips off the plate at the bottom

Assumptions: steady operating conditions exist, plate is isothermal, $\rho_v \ll \rho_l$

The properties of water at the saturation temperature of 100°C are:

$$T_{sat} = 100^\circ\text{C}, h_{lv} = 2257 \text{ kJ/kg}, \rho_v = 0.6 \text{ kg/m}^3$$

The properties of liquid water at the film temperature 90°C:

$$\text{Pr} = 1.9628, \rho_l = 965.3 \text{ kg/m}^3, \mu_l = 0.315 \times 10^{-3} \text{ Pa s}, C_{pl} = 4.206 \text{ kJ/kg K}, \\ k_l = 0.675 \text{ W/m K}$$

$$h'_{fg} = h_{fg} + 0.68C_{p,l}(T_{sat} - T_w) = 2.314 \times 10^6 \text{ J/kg}$$

Let us start with laminar flow model

$$\bar{h}_L = 0.943 \left(\frac{g \rho_L (\rho_L - \rho_v) h_{fg} k_l^3}{4 \mu_L (T_{sat} - T_w) L} \right)^{\frac{1}{4}} = 2656.2 \text{ W / m}^2 \text{ K}$$

$$\dot{Q} = \bar{h}_L A_s (T_{sat} - T_w) = 2656.2 \times 2 \times 3 \times (100 - 80) = 307.464 \text{ kW}$$

$$\dot{Q} = \dot{m} h'_{fg}$$

$$\Rightarrow \dot{m} = 0.1329 \text{ kg/s}$$

$$\text{Re} = \frac{4\Gamma}{\mu_L} = \frac{4}{\mu_L} \left(\frac{\dot{m}}{b} \right) = \frac{4}{0.315 \times 10^{-3}} \left(\frac{0.1329}{3} \right) = 562.5$$

$$\boxed{\text{Re} > 30}$$

Hence, the flow is not laminar. Try wavy-laminar flow

$$\text{Re}_{\text{vert,wavy}} = \left[4.81 + \frac{3.70 L k_l (T_{\text{sat}} - T_w)}{\mu_l h'_{fg}} \left(\frac{g}{\nu_l^2} \right)^{\frac{1}{3}} \right]^{0.82}$$

$$= \left[4.81 + \frac{3.70 \times 3 \times 0.675 (100 - 90)}{\mu_l h'_{fg}} \left(\frac{9.8}{(0.326 \times 10^{-6})^2} \right)^{\frac{1}{3}} \right]^{0.82}$$

$$\text{Re} = 1287, \text{ ie., } < 1800$$

Hence, the flow is wavy-laminar.

$$h_{\text{vert,wavy}} = \frac{\text{Re } k_l}{1.08 \text{Re}^{1.22} - 5.2} \left(\frac{g}{\nu_l^2} \right)^{\frac{1}{3}} \quad \bar{h} = 5848 \text{ W / m}^2 \text{ K}$$

$$\dot{Q} = \bar{h}_L A_s (T_{\text{sat}} - T_w) = 5848 \times 2 \times 3 \times (100 - 80) = 702 \text{ kW}$$

$$\dot{Q} = \dot{m} h'_{fg}$$

$$\Rightarrow \dot{m} = 0.303 \text{ kg/s}$$

Steam will condense on the surface at a rate of 303 grams per second.